Chapter 14

Finale: what we would like to know

14.1 Individual versus uniform eventual positivity for self-adjoint semigroups

Corollary 13.2.2 shows that for a σ-finite measure space (Ω, ν), a vector $0 \le u \in L^2(\Omega, \nu)$, and a self-adjoint semigroup generator A on $L^2(\Omega, \mu)$, individual eventual positivity of $(e^{tA})_{t\ge 0}$ with respect to u is equivalent to uniform eventual positivity with respect to $u \otimes u$ if, for some $m \in \mathbb{N}$, the domain $\operatorname{dom}(A^m)$ is contained in the the principal ideal $L^2(\Omega, \nu)_u$. In fact, the proof of Corollary 13.2.2 shows that the same equivalence stays true under the slightly weaker assumption that $\operatorname{rg} e^{t_0 A} \subseteq L^2(\Omega, \nu)_u$ for some $t_0 > 0$ and that u is strictly positive almost everywhere.

On the other hand, Examples 11.1.2 and 13.1.2 show that individual and eventual uniform eventually positivity are in general not equivalent without appropriate "smoothing assumptions". However, the spaces in those counterexamples are quite far from being L^2-spaces, and the semigroups appear to behave very "symmetrically", which would suggest that one could modify them to obtain self-adjoint counterexamples on L^2. This motivates the following question.

Open Problem 14.1.1. Let $(e^{tA})_{t\ge 0}$ be a real C_0-semigroup on $L^2(\Omega, \nu)$ for a σ-finite measure space (Ω, ν) and assume that A is self-adjoint.

(a) Are individual and uniform eventual positivity of $(e^{tA})_{t\ge 0}$ with respect to 0 equivalent?

(b) Let $0 \le u \in L^2(\Omega, \nu)$ be strictly positive almost everywhere. Is individual eventual positivity of $(e^{tA})_{t\ge 0}$ with respect to u equivalent to uniform eventual positivity with respect to $u \otimes u$?

If these questions turn out to be too difficult to answer in general, can they be answered under the additional assumption that A has compact resolvent?

The lecturers are not aware of a promising approach to prove a positive result, nor do they have good suggestions on how to construct counterexamples. In [Glü16, Theorem

10.2.1], the result of Corollary 13.2.2 was shown by quite different methods, but they also heavily rely on the assumption $\operatorname{dom}(A^m) \subseteq L^2(\Omega,\mu)_u$.

14.2 Eventual positivity via forms

The Beurling–Deny criterion (Theorem 5.1.7 and Corollary 10.3.4) is extremely useful to check positivity for semigroups whose generator stems from a sesquilinear form on an L^2-space, as evidenced by many examples where we employed these criteria throughout the ISEM 29. Likewise, having a criterion for eventual positivity of semigroups (or resolvents) in terms of sesquilinear forms would likely be extremely useful in studying eventual positivity for many further PDEs. Thus it is natural to ask the following:

Open Problem 14.2.1. Let $(e^{tA})_{t\geq 0}$ be a real C_0-semigroup on $L^2(\Omega,\nu)$ for a σ-finite measure space and assume that the generator A is associated to a sesquilinear form $\mathfrak{a}\colon V\times V\to\mathbb{C}$ that satisfies the assumptions of Theorem 5.1.4 (for $H = L^2(\Omega,\nu)$).

Characterise individual/uniform eventual positivity of $(e^{tA})_{t\geq 0}$ (with respect to 0 or respectively a non-zero vector/operator) in terms of the form domain V and the form $\mathfrak{a}$.

There are a number of reasons to believe that this problem is very hard:

(a) In finite dimensions, where A is a simply a matrix, there are no issues with the domains of forms and operators, and the connection between a form and its associated matrix is very simple. Therefore, a complete answer to Open Problem 14.2.1 would likely include a characterisation of eventual positivity for matrix semigroups in terms of the entries of the generator. However, no such criterion is currently known. Even for matrices, criteria to check eventual positivity of $(e^{tA})_{t\geq 0}$ rely on the analysing the spectral bound of A and the associated eigenspaces of A and A^{T}.

Nevertheless, this does not exclude the possibility of finding such characterisations for important classes of differential operators. Such operators, and their associated forms, exhibit a "local" type of behaviour that does not occur in finite dimensions, except for diagonal matrices.

(b) The proof of the Beurling–Deny criterion in Theorem 5.1.7 relies heavily on the behaviour of the resolvent $\mathcal{R}(\lambda, A)$ for large real numbers λ. This corresponds to the behaviour of the semigroup for small times.

(c) There are differential operators A on bounded domains $\Omega\subseteq\mathbb{R}^n$ for which the semigroup $(e^{tA})_{t\geq 0}$ can be eventually positive or not, depending on the specific geometry of Ω. In the setting of Theorem 13.1.1, it is necessary for eventual positivity that the spectral bound is a strictly dominant eigenvalue, and that the corresponding eigenspace is spanned by a positive eigenfunction. The latter is a very subtle matter for higher-order elliptic operators. For instance, we refer to [SS20, Section 3] for an intriguing discussion of positivity of the eigenfunctions corresponding to the spectral bound for the biharmonic operator Δ^2 with clamped boundary conditions[1] for family of annuli in $\mathbb{R}^2$. Any criteria for eventual positivity in terms of the associated form $\mathfrak{a}$ would thus likely establish a rather direct link between the geometry of Ω and properties of the form $\mathfrak{a}$ and its form domain V.

The more "direct" or "explicit" the eventual positivity criterion, the more clearly the geometry of Ω would have to be encoded in $\mathfrak{a}$ and V.

14.3 Spectral bound vs. growth bound on L^p

Theorem 11.4.4 shows the equality $\mathrm{s}(A) = \omega_0(A)$ of the spectral and the growth bound for eventually positive semigroups $(e^{tA})_{t\ge 0}$ if the underlying space is L^1 or $\mathrm{C}_0(\Omega)$. The same result is known to hold on L^2-spaces [DGK16b, Theorem 7.8(i)]; there it follows from the famous **Gearhart–Prüss theorem** (see e.g. [EN00, Theorem V.1.11]) and the resolvent estimate in Exercise 11.3(b). On L^p-spaces for general $p \in [1,\infty]$, the equality $\mathrm{s}(A) = \omega_0(A)$ was shown for uniformly eventually positive semigroups by Vogt [Vog22]. Even for positive semigroups this proof is simpler than earlier proofs on L^p. Unfortunately, the argument cannot simply be transferred to individually eventually positive semigroups, which leaves the following question open.

Open Problem 14.3.1. Let (Ω,ν) be a σ-finite measure space let $p \in [1,\infty]$ and let $(e^{tA})_{t\ge 0}$ be a C_0-semigroup on $L^p(\Omega,\nu)$ that is individually eventually positive with respect to 0 (and real, if this is of any help). Does it follow that $\mathrm{s}(A) = \omega_0(A)$?

As pointed out above, the answer is known to be positive if $p = 1$ or $p = 2$. For $p = \infty$ the answer is also positive for any of the following independent reasons: on one hand, $L^\infty(\Omega,\mu)$ is, as a Banach lattice, isomorphic to $\mathrm{C}(K)$ for an appropriate compact Hausdorff space K due to Kakutani's representation theorem of AM-spaces with unit [MN91, Theorem 2.1.3]; so the result is a special case of Theorem 11.4.4(b). On the other hand, it was shown by Lotz that a C_0-semigroup $(e^{tA})_{t\ge 0}$ on $L^\infty(\Omega,\mu)$ is always continuous with respect to the operator norm on the entire time interval $[0,\infty)$ (see [Lot85] or [AGG⁺86, Theorem A-II.3.6]), so the equality $\mathrm{s}(A) = \omega_0(A)$ follows from Theorem 12.2.2(a).

14.4 A priori lower bounds

Theorem 9.1.1 gives necessary conditions for uniform eventual positivity of resolvents in terms of the resolvents bounds $\pm\mathcal{R}(\nu, A) \preceq u \otimes \varphi$ for all $\nu \in \rho(A) \cap \mathbb{R}$. It turns out that this theorem is not optimal. If the vector u and the functional φ are comparable to eigenvectors of A and A^*, the same conclusion can be shown even under weaker assumptions:

Theorem 14.4.1. *Let $A\colon E \supseteq \operatorname{dom}(A) \to E$ be a closed, densely defined, and real operator on a complex Banach lattice E. Assume $\lambda_0 \in \sigma(A) \cap \mathbb{R}$ is an isolated spectral value of A. Let $u \in E_+$, let $\varphi \in E'_+$ be strictly positive, and assume:*

[1] See the Notes for Chapter 9.

(0) $\ker(\lambda_0 - A)$ *is spanned by a vector* v *that satisfies* $v \succeq u$, *and* $\ker(\lambda_0 - A)$ *contains a vector* ψ *that satisfies* $\psi \succeq \varphi$.

(1) $\operatorname{dom}(A^{m_1}) \subseteq E_u$ *for some* $m_1 \in \mathbb{N}$.

(2) $\operatorname{dom}\big((A')^{m_2}\big) \subseteq (E')_\varphi$ *for some* $m_2 \in \mathbb{N}$.

(3) *There are real numbers* $\mu_-, \mu_+ \in \rho(A)$ *that satisfy* $\mu_- < \lambda_0 < \mu_+$ *as well as the estimates* $\mathcal{R}(\mu_-, A) \preceq u \otimes \varphi$ *and* $\mathcal{R}(\mu_+, A) \succeq -u \otimes \varphi$.

Then the resolvent $\mathcal{R}(\cdot, A)$ *is uniformly eventually positive and negative with respect to* $u \otimes \varphi$ *at* λ_0 *and thus,* $\pm\mathcal{R}(\nu, A) \preceq u \otimes \varphi$ *for all* $\nu \in \rho(A) \cap \mathbb{R}$ *by Theorem 9.1.1.*

This is (and even a bit more) is proved in [AG22a, Theorem 1.2]. The most important thing here is what happens in condition (3): the assumption on the right of λ_0 is merely $\mathcal{R}(\mu_+, A) \succeq -u \otimes \varphi$ – carefully note the minus sign on the right hand side. Likewise, the assumption on the left of λ_0 is merely $\mathcal{R}(\mu_-, A) \preceq u \otimes \varphi$, which is a priori much weaker than the conclusion of uniform eventual negativity with respect to $u \otimes \varphi$. Theorem 14.4.1 is useful to prove or disprove uniform eventual positivity of resolvents in various concrete examples of differential operators.

At the same time, it raises two questions. The first one is inspired by the observation that assumption (0) was not needed in Theorem 9.1.1.

Open Problem 14.4.2. If one drops assumption (0) in Theorem 14.4.1, does the last conclusion of the theorem ($\pm\mathcal{R}(\nu, A) \preceq u \otimes \varphi$ for all $\nu \in \rho(A) \cap \mathbb{R}$) still remain true?

The remaining assumptions in Theorem 14.4.1 are certainly not enough to imply the uniform eventual positivity and negativity of $\mathcal{R}(\cdot, A)$ with respect to $u \otimes \varphi$ if one drops assumption (0), since then the information provided by (3) is too little. (To find an explicit counterexample, use the Neumann Laplace operator on $L^2(0,1)$ and choose $u = \varphi$ to be functions in $L^2(0,1)$ that are not in $L^\infty(0,1)$.)

The second problem is inspired by the question of how the assumptions $\mathcal{R}(\mu_-, A) \preceq u \otimes \varphi$ and $\mathcal{R}(\mu_+, A) \succeq -u \otimes \varphi$ in Theorem 14.4.1(3) can be checked in concrete examples where A is a differential operator. Here, a quite surprising phenomenon occurs: it has been shown by PDE methods that at least the second of those two estimates is true for a variety of higher order elliptic operators. More specifically, the situation is as follows:

For a "sufficiently nice" differential operator A on a bounded domain $\Omega \subseteq \mathbb{R}^n$ one can write the resolvent $\mathcal{R}(\lambda, A)$ at $\lambda \in \mathbb{R} \cap \rho(A)$ as an integral operator, where the integral kernel $G_\lambda : \Omega \times \Omega \to \mathbb{R}$ is called the *Green's function* of A (at the point λ). Upper or lower estimates for G_λ are equivalent to upper or lower estimates of the resolvent operator $\mathcal{R}(\lambda, A)$. In higher dimensions, the Green's function usually explodes at the diagonal of $\Omega \otimes \Omega$. Remarkably, however, if $\lambda > \operatorname{s}(A)$, one can show for many differential operators that the negative values of G_λ are actually bounded – i.e. the singularity of G has a positive sign, although the resolvent is not necessarily positive. Similarly, one often has very good control over the negative part of G_λ close to the boundary of Ω, which is also useful to establish estimates of the type $\mathcal{R}(\mu_+, A) \succeq -u \otimes \varphi$.

Positivity (and negativity) properties of the Green's function of higher-order elliptic operators have been extensively studied over the past few decades. In particular, analysis of the 'smallness' of regions of negativity can be found in [GR10, Theorem 1], with further refinements in [GRS11, Theorem 1]; see also [GS02, Theorem 1.2], [DMS05, Theorem 1.5], and [Pul15, Theorem 4.1] on the subject of upper and lower estimates.

However, the lecturers find themselves to be completely oblivious regarding any abstract operator theoretic explanation for this kind of behaviour. This motivates the following problem.

Open Problem 14.4.3. For an operator $A\colon E \supseteq \operatorname{dom}(A) \to E$ on a complex Banach lattice E with $\operatorname{s}(A) \in \sigma(A)$ and elements $u \in E_+$ and $\varphi \in E'_+$, find general criteria to ensure

$$\mathcal{R}(\lambda, A) \succeq -u \otimes \varphi \qquad \forall\, \lambda > \operatorname{s}(A).$$

In other words, the problem is to give an operator theoretic explanation why the negative part of the Green's function G_λ of a higher-order elliptic differential operator can often be much better controlled than its positive part.

The bi-Laplacian with Wentzell boundary conditions on bounded domains with Lipschitz boundary was analysed by Denk, Kunze, and Ploß in [DKP21]. They show that the operator generates a bounded C_0-semigroup that is uniformly eventually positive under appropriate assumptions. In fact, using Theorem 7.3.6, one can even show (for low dimensions) that the resolvent is individually eventually positive (and negative) at the spectral bound; see [Aro23, Theorem 5.2.4] for details. On the other hand, we do not know whether the resolvent is uniformly eventually positive, as the corresponding resolvent bounds are unknown, cf. [Aro23, Remark 5.2.5].

14.5 Criteria for eventual positivity with respect to 0

All our characterisation results for eventual positivity (both for resolvents and for semigroups) always refer to individual eventual positivity with respect to a quasi-interior point $u \in E_+$ or with respect to an operator $u \otimes \varphi$ for a quasi-interior point $u \in E_+$ and a strictly positive functional $\varphi \in E'_+$. For eventual positivity with respect to 0, we have only discussed – and in fact only know – necessary criteria in terms of spectral properties of the operator A. We thus ask:

Open Problem 14.5.1. For a (real, if it helps) closed operator $A\colon E \supseteq \operatorname{dom}(A) \to E$ on a complex Banach lattice E with spectral bound $\operatorname{s}(A) \in \mathbb{R}$, what are characterisations for...

(a) ...individual or uniform eventual positivity of $\mathcal{R}(\cdot, A)$ at $\operatorname{s}(A)$ with respect to 0?

(b) ...individual or uniform eventual positivity of $(e^{tA})_{t\geq 0}$ with respect to 0 (under the assumption that A generates a C_0-semigroup)?

The lecturers consider it likely that the question is far too difficult to have a comprehensive answer in full generality. But even in many special cases, for instance if A

has compact resolvent, such a characterisation would probably provide very interesting additional insights into the structure of eventual positivity.

A very closely related question is the following.

Open Problem 14.5.2. Let $(e^{tA})_{t\geq 0}$ be a (real, if it helps) C_0-semigroup on a complex Banach lattice E and assume that $\mathrm{s}(A) \in \sigma(A)$. Clarify the relation between individual (uniform) eventual positivity of $(e^{tA})_{t\geq 0}$ with respect to 0 and individual (uniform) eventual positivity of $\mathcal{R}(\cdot, A)$ at $\mathrm{s}(A)$ with respect to 0.

14.6 (Local) eventual positivity on unbounded domains

Observant readers may have noticed that all examples of eventual positivity discussed in these lecture notes involve operators acting on function spaces over bounded domains. The underlying reason is that most available results on eventual positivity – and, in particular, sufficient conditions for it – require the spectral bound $\mathrm{s}(A)$ to be a pole of the resolvent and a geometrically simple eigenvalue. This situation commonly arises for differential operators on bounded domains, where compact embeddings between suitable Sobolev spaces often imply compactness of the resolvent under mild assumptions.

By contrast, for operators on unbounded domains the resolvent is typically not compact, and poles of the resolvent occur only in rather exceptional cases; see, for instance, [AGRT22, Remark 3.9]. This naturally raises the question of what can be said in the unbounded setting. In fact, the first examples where eventual positivity – or more precisely, a local version thereof – was observed, is the biharmonic heat equation $\dot{u}(t) = -\Delta^2 u$ on the whole space $\mathbb{R}^d$ [FGG08, GG08]. It is hardly an exaggeration to say that the methods presented throughout the ISEM 29 to prove eventual positivity completely break down if the spectral values are not eigenvalues.

This stands in sharp contrast to certain methods to prove positivity (rather than only eventual positivity): in particular, the Beurling–Deny criterion in Theorem 5.1.7 and Corollary 10.3.4 neither requires boundedness of the spatial domain nor depends on detailed spectral information, let alone the existence of positive eigenvectors.

Local eventual positivity of the biharmonic heat equations was shown in [FGG08, GG08] and, for higher-order differential operators on $\mathbb{R}^n$, in [FF19], by means of explicit estimates for the convolution kernel of the associated semigroup – the so-called **biharmonic** or **polyharmonic heat kernel**. An alternative and more qualitative approach, based on Fourier transform methods, can be found in [DGM23]. There, these techniques are further combined with spectral-theoretic arguments to obtain local eventual positivity for the biharmonic heat equation on infinite cylinders in $\mathbb{R}^{n+1}$.

Nevertheless, all currently available methods appear to rely heavily on the relatively simple structure of the differential operator and on the specific geometry of the underlying domain. Even seemingly modest extensions – such as considering the biharmonic heat equation on a half-space or on a quadrant in $\mathbb{R}^2$ – require the development and careful adaptation of substantial, largely ad hoc, computational machinery. In this light, abstract operator-theoretic results would be highly desirable, both to alleviate the computational burden and to sharpen our conceptual understanding of when (local) eventual positivity can be expected on unbounded domains.

Open Problem 14.6.1. Develop general tools to analyse when the semigroup generated by a differential operator on an unbounded domain is (locally) eventually positive.

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